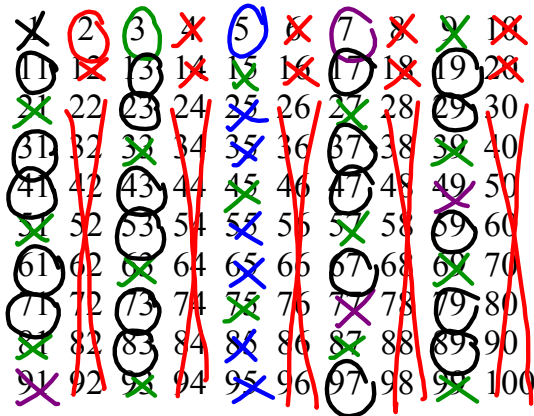4.2: Prime and Composite Numbers

Definition: A prime number is a number with exactly two distinct positive factors, namely 1 and themselves.

Definition: A composite number is a number with more than two distinct positive factors:

Is 1 a prime number or a composite number?

Neither, it has only 1 factor.

Find which numbers are prime in the set {1, 2, ..., 100}

This is known as the Sieve of Eratosthenes.

Theorem: If n is composite, then it has a prime factor p with the property that $p^2 \leq n$.

In other words, to see if a number is prime, we need only check all of the possible <u>prime</u> factors up to its square root.

Proof:

Let p be the smallest divisor of n that is not 1. Then there is a number k such that $pk = n$. $(k = \frac{n}{p})$. $k \geq p$ since p is the smallest divisor. So,

$$n = pk \geq p \cdot p = p^2, \text{ so } p^2 \leq n. \blacksquare$$

Example: List the factors of 28. Is 28 prime or composite?

$1, 2, 4, 7, 14, 28$ composite

Is 301 prime?

$\sqrt{301} \approx 17.35$

~~2~~   ⑦   17

~~3~~   11

~~5~~   13 $7 | 301$, so 301 is composite
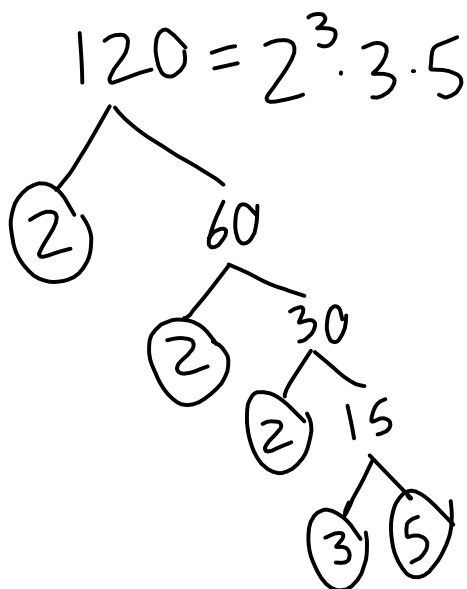
Is 307 prime?

$\sqrt{307} \approx 17.52$

~~2~~   ~~7~~   ~~17~~

~~3~~   ~~11~~

~~5~~   ~~13~~   307 is prime

Fundamental Theorem of Arithmetic: Each composite number can be written as a product of primes in exactly one way (ignoring the order of the factors).

Definition: This product described above is known as the<u>prime factorization</u> of a number.

Example: What is the prime factorization of 120?

$$120 = 2^3 \cdot 3 \cdot 5$$

Example: What is the prime factorization of 270?

$$270 = 2 \cdot 3^3 \cdot 5$$